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Investigation of the Flare Maneuver Using Optical Tau

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Airline transport operations are carried out in a wide range of visual and instrument meteorological conditions. However, the pilot can choose to land the aircraft manually using the visual cues available for all but the most limiting of degraded visibilities. How this is achieved may seem rather obvious, but has challenged researchers for some time. Optical flow theory offers a solution: pilots detect motion from the surfaces over which they move. In a relatively recent incarnation, flow theory transforms motion into the temporal, time-to-contact parameter, tau. Research conducted at Liverpool has applied this theory to low-level helicopter flight. The present paper extends the application to the fixed-wing flare. Flight simulation results show that tau-guidance strategies exist for this maneuver. It is shown that, as expected from tau theory, the pilot-selected values of the rate of change of tau with time, tau-dot, and the tau-guide coupling constant, directly influence the acceptability of the touchdown rate achieved. Degraded visual environments are shown, under certain circumstances, to cause a breakdown in the tau relationships observed. Potential uses of these results are presented in terms of application to future pilot vision aids, which is the planned next stage of this work.

Nomenclature

a = constant value of τ
C = integration constant
c = constant value of rate of change of τ
D_L = perceived distance to landing point
$D_{R'}$ = perceived distance to some point ahead of the landing point
d = distance between observer and image projection plane
h = current value of height gap to be closed
h_0 = start height for flare maneuver
K = constant of proportionality for tau coupling
k = constant of proportionality for intrinsic tau-guidance strategies
T = total time of a maneuver
T_G = total time of a maneuver where motion is coupled to a tau guide
t = current time during a maneuver where motion may be coupled to a tau guide
t_n = normalized time, t/T_G
t_0 = small positive start time for motion coupled to τ_g
u, v = general gap variables used to show that tau coupling implies a power law relationship
V_{stall} = aircraft indicated airspeed at the stall
x, y = externally perceived spatial variables used for external tau coupling
x_m = minimum value of x achieved
θ_{appr} = aircraft pitch angle for steady approach
θ_{td} = aircraft pitch angle at touchdown
τ = time to contact surface at current closure rate
$\dot{\tau}$ = rate of change of time to contact surface at current closure rate
τ_G = general intrinsic tau guide
τ_g = special case of general intrinsic tau guide corresponding with motion from rest to goal only

I. Introduction

IN the highly digitized and automated world of fixed-wing airline transport operations, given a suitably equipped aircraft and airfield, today's airline pilot could choose to fly an entire sector on autopilot. Of course, not all aircraft or airfields are suitably equipped. In such cases, the pilot will have to fly some or all portions of the sector manually using visual references. This is particularly true during the flare and touchdown phases of a landing. These phases of flight, by their very nature, occur near to or on the Earth's surface. Statistics show that it is within these phases of flight that fatal accidents are most likely to occur [1]. When flying visually, the pilot must rely on the motion information received from the view of the outside world to continuously correct heading, height, and horizontal and vertical speed to bring the aircraft into contact with the runway surface at a particular location and at a rate acceptable to any passengers, the aircraft operator, and the aircraft manufacturer. (These are not the only cues available, of course; for example, the vestibular systems provides additional information about the accelerations that the aircraft is experiencing. However, pilots are trained to be wary of these cues as they can be misleading when not backed up by outside world visual flight information or that obtained from instruments.) Any aircraft motion that deviates from that which is desired must be corrected and this task must be achieved within the constraints of both the aircraft dynamics and the view available through the cockpit windscreen.

The view from the cockpit is not always necessarily as comprehensive as the pilot would expect. In an air transport operational environment, manual landings tend to be carried out down to Category I minima. (The final decision to continue an approach is made at 200 ft and visual range on runway must be greater than or equal to 1800 ft [2]. For more restrictive visual conditions, monitoring of automatic landing systems by the pilot is the usual practice.) This provides the pilot with some 8 s of look-ahead time and perhaps double that before touchdown. In this time, the pilot must look up from the instruments, ensure that s/he has visual contact with sufficient cues to legally make the landing, establish the orientation of the aircraft in relation to the outside world, and prepare to make the appropriate control inputs to flare and land the aircraft.

The question therefore arises as to the manner in which the pilot is able to control the motion of the aircraft with the restricted view available (restricted by both the window frames and perhaps by the weather conditions as well). From an engineering perspective of the

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flare, the pilot should aim to bring the aircraft height above ground to zero at a particular location as the rate of change of height also becomes close to zero. The pilot performs this task by transforming the perceived aircraft motion from the optical frame of reference into the inertial frame of reference and applying feedback to minimize errors between the commanded and perceived motion [3]. This leads to a further question about what information the pilot uses to perceive his or her motion. From a psychological perspective, a number of different descriptions of the pilot's perception of his/her motion are available. Constructivist psychologists would say that the pilot cannot perceive his/her world directly and must therefore interpret the aircraft motion via the retinal image through knowledge acquired through learning during training [4]. Gestalt psychologists would say that the pilot organizes the available stimuli into patterns and responds to relationships among those stimuli to enable the aircraft to be guided to a safe touchdown [5]. Ecological psychologists, in contrast with the constructivists, would emphasize that the pilot perceives the aircraft and hence his/her own motion (known as ego-motion) directly from the optic flow of surfaces in the field of vision [6]. In this case, it is postulated that optic flow specifies how the observer is moving in relation to his/her environment.

The research described in this paper has been guided by the ecological approach to motion perception using optic flow as a start point. Optic flow rate can provide the pilot with information on ground speed in body-scaled units (eye-heights per second [7]) or surface slant [8]. Differential motion parallax can be used by pilots for way finding in a cluttered environment [9]. Lee developed the theory of optic flow and direct motion perception by introducing an optical variable that gives this paper its name, tau: the time to contact or close to an obstacle or surface at the current closure rate [10]. Tau theory provides a framework for perceiving motion in terms of time and serves as a universal variable for controlling motion-gaps. Evidence for the universal existence of tau has been offered in the form of hummingbirds docking on a feeder [11], bats echo locating prey [12], and automobile drivers braking to a halt [13]. In a more relevant context to this paper, evidence for tau-guidance strategies has been demonstrated in simulated helicopter maneuvering [3]. Here, it is shown that when helicopter pilots fly stopping maneuvers close to the ground, there is a close correlation between the motion-tau (instantaneous time to reach the stop point) and a pilot-generated tau-guide that can follow constant deceleration or acceleration laws. It is postulated that the correlation is so good that the tau model of pilot visual perception and motion is suitable for extension to other flight maneuvers. This paper provides the first evidence of tau-guidance strategies being used in fixed-wing aircraft flight for the landing flare maneuver.

The ultimate aim of the research is to develop guidelines for display design through a three-stage approach. The first is to understand the optical cues that the pilot uses to guide the motion of the aircraft. The second stage is to then degrade the visual environment until those optical cues are no longer available, to establish a better understanding about how the pilot copes in such a scenario. The final stage of the research project, entitled prospective sky guides (PSG), is to then design displays that recover the visual cues that are necessary for the pilot to achieve safe flight. This paper reports on elements of the first two stages of this approach for the landing flare maneuver.

II. Background to the Research Effort

A. Visual Perception and the Optical Parameter "Tau"

The ecological approach to perception and the guidance of animal motion, pioneered by J. J. Gibson, has its foundations in aviation. Gibson investigated the use of pictures, and motion pictures in particular, for selection and training of aircrew for the United States Army Air Corps [14]. Gibson was particularly attracted to the motion picture as a form of training aid due to the additional information that was available to the observer from the motion displayed by the film. Gibson later hypothesized that this extra information came from the optic flowfield (the way in which individual points in the scene move from moment to moment) that the motion caused. To help to illustrate this concept, Fig. 1 shows the optic flow available to a pilot flying straight and level over an airfield [6]. In his later work, Gibson cites the approach and land case as one example of how a pilot uses the optic flow available to him/her to control an aircraft [6,15]

Lee, a student of Gibson, further developed the theory of optic flow and introduced an optical parameter, tau, the time to contact a surface or object a distance x away at the current closure rate ($\dot{x}$), as per Eq. (1):

$$\tau_x = \frac{x}{\dot{x}} \qquad (1)$$

Tau theory has been continuously developed and Lee [10] provides the principal tenets of tau theory for guiding movement. In the context of pilot control of an aircraft flying visually, these may be interpreted as follows:

1) A central task in guiding movement is to control the closure of spatial gaps between the aircraft's current state (or position or orientation) and its desired state (or position or orientation). The pilot will effect this closure using the control inceptors available to him, e.g., stick, throttle, pedals, etc.

2) The closure of gaps requires the sensing of the visual input array.

3) The tau of each spatial gap (the time-to-closure of the gap at its current closure rate) is what is constantly sensed and controlled to guide the movement.

4) A principal method of motion guidance is by coupling the taus of different gaps, that is, keeping the taus in constant ratio.

For the theory to work, of course, tau information must be available to and sensed by the pilot (preceding tenets 2 and 3). Lee [16] provides detailed information on how the sensing of tau might be achieved. In brief, if there is a power law relation between an external and a sensory motion gap, then the tau of the external motion gap can be sensed directly as a multiple of the tau of the sensory gap. Appendix A shows how the tau of a vertical movement of the aircraft can be sensed by the pilot. It should be noted that, at this stage, the actual visual "cues" sensed by the pilot to close the gaps are not in themselves under investigation; in a good visual environment, there are likely to be a variety of components in the optical flowfield providing a degree of redundancy, and therefore sources for tau coupling, to the pilot.

The earliest hypothesis concerning the use of tau in controlling the closure of motion was that during the deceleration phase to an obstacle/surface, the rate of change of tau is maintained constant [13]:

$$\dot{\tau} = c \qquad (2)$$

Using the flare to touchdown as an example, x in Eq. (1) might be hypothesized to be aircraft height and $\dot{x}$, aircraft vertical speed. Figure 2 shows how the theoretical aircraft flare trajectory, vertical

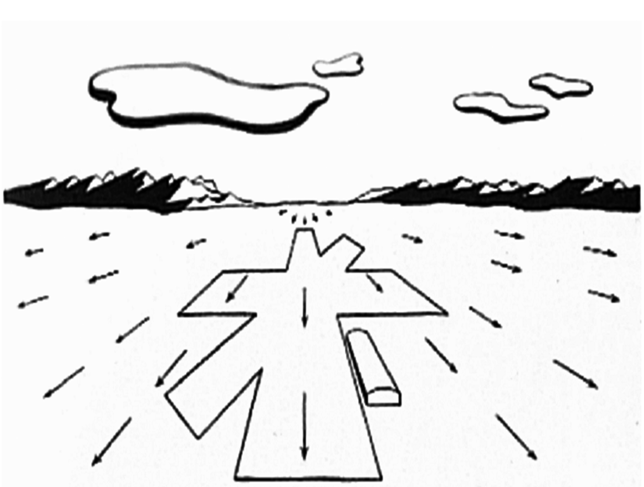


Fig. 1 Optic flowfield observed by pilot flying straight and level over an airfield [6].

velocity and height tau, $\dot{\tau}_h$, will vary depending upon the (constant) value of c that the pilot selects for $\dot{\tau}_h$, assuming that the flare is commenced at 50 ft above ground level (AGL). The method used to construct these curves can be found in Appendix B.

If c is maintained at less than or equal to 0.5, arresting the vertical rate above or at the ground is assured [17]. If c is held at a value greater than 0.5, then the obstacle will be reached with some residual velocity (a firm touchdown). If c is held at 1.0, a constant velocity of approach is maintained. Values of c greater than 1.0 result in an accelerative approach to the surface.

Maintaining a constant rate of change of the tau of a particular gap is valid for single gap closures but there are more generally going to be occasions where multiple gaps need to be closed simultaneously. This is achieved, it is postulated, by coupling the taus of the gaps of interest, i.e., keeping them in constant ratio (preceding tenet 4). The coupling of the taus of gaps may be extrinsic or intrinsic. Extrinsic coupling uses two external parameters that can be sensed by the pilot. In this case,

$$\tau_x = K\tau_y \tag{3}$$

where x and y are the externally perceived spatial variables (e.g., height of the aircraft above ground and the distance to go to the desired touchdown point), K is constant, and τ_x and τ_y vary with time.

Intrinsic coupling keeps one externally perceived variable in constant ratio with an intrinsic tau guide that, it is hypothesized, is generated by a bodily process, e.g., electrical charge flowing in the brain. For a motion to couple onto the original version of this intrinsic tau guide,

$$\tau_x = k\tau_g \tag{4}$$

where x is the externally perceived spatial variable, k is constant, τ_g is the intrinsic tau guide, and both τ_x and τ_g vary with time. τ_g is given as [3,10]

$$\tau_g = \frac{1}{2}\left(t - \frac{T^2}{t}\right) \tag{5}$$

where $t_0 < t \leq T$. $t = 0$ is excluded in this case as $\tau_g = \infty$.

The original model, given in Eq. (5), was valid for guiding motion of an object accelerating from rest and stopping at a goal [17]. The intrinsic tau-guide model has since been developed further into the *general intrinsic* tau-guide model, for guiding the motion of an object that is approaching or receding from a destination and that starts at rest or starts with some initial velocity [17]. This general tau-guide is given as [17]

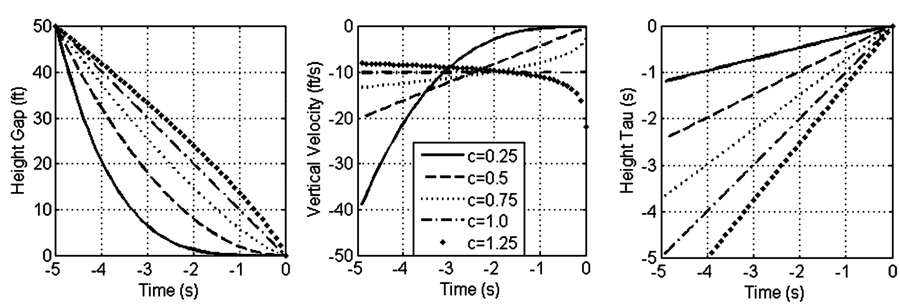
$$\tau_G = \frac{t(T + t)}{T + 2t} \tag{6}$$

where $-T \leq t \leq 0$. For an external spatial variable x to tau couple onto the general intrinsic tau guide,

$$\tau_x = k\tau_G \tag{7}$$

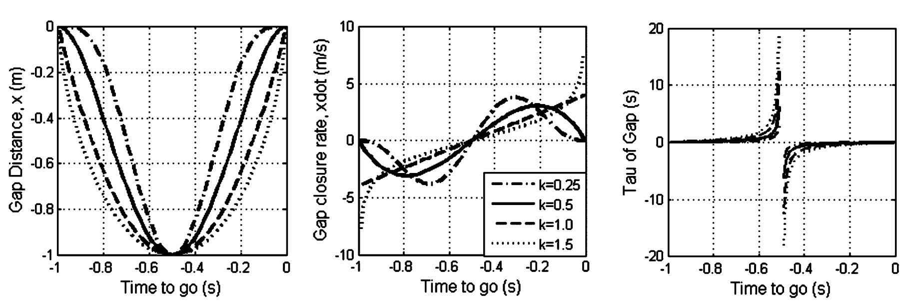
where τ_x is the tau of the spatial variable x and k is the coupling constant. Both τ_x and τ_G vary with time. When coupled onto such a guide, an object in motion will follow one of the theoretical normalized motion profiles shown in Fig. 3, taken from [17]. A summary of the background used to plot the curves of Fig. 3 is given in Appendix C. τ_g, it turns out, is a special case of τ_G, corresponding to the second half of the motion generated by τ_G [17] (see Appendix D). Inspection of the τ_G-coupled motion profiles reveals that the value of k selected will provide differing responses when approaching a target surface or object (from time to go −0.5 to 0.0). A value of $k < 1.0$ results in an acceleration–deceleration motion. As k approaches 1.0, the deceleration phase of the motion starts at an increasingly later time. If $k = 1.0$, the resulting motion is performed under constant acceleration. The body under motion reaches the target with some residual velocity (just as an aircraft touches down with a finite vertical velocity). If a value of $k > 1.0$ is selected, then the object continues to accelerate towards the target. The profiles described by the normalized version of Eq. (6) and illustrated in Fig. 3 can be used as a template when identifying motions that are potentially guided using this tau strategy.

To date, evidence for the existence of the general intrinsic tau guide has been demonstrated in the control of oral suction by newborn babies, the vocalizing of a scale and other musical intervals, and the guidance of gaze direction when tracking a moving target [17].



a) b) c)

Fig. 2 Theoretical flare trajectory data for constant height tau-dot.



a) b) c)

Fig. 3 Theoretical normalized observer motion when coupled to the general intrinsic tau guide.

B. The Control of Aircraft Motion Using Tau

The temporal approach to motion guidance is a compelling one and there is growing evidence in the natural and man-made world that motion control is fundamentally driven by the tau of gaps. The question remains, however, is there evidence for tau-based strategies in the guidance of aircraft motion? If external motion tau relationships can be identified for a particular maneuver, then the possibility exists that that sensory tau relationship is being used to control the motion. The first evidence of tau coupling being used to guide aircraft motion has been for simulated flight in rotary wing aircraft [3]. It was shown that when helicopter pilots fly stopping maneuvers close to the ground, there is a close correlation between the motion-tau (instantaneous time to reach the stop point) and a pilot-generated tau guide that can follow the constant $\dot{\tau}$ laws given by Eqs. (2) and (5). Furthermore, the data suggest that the correlation is so strong that the tau model of pilot visual perception and motion is likely to be suitable for extension to other flight maneuvers and that, by inference, optical tau and $\dot{\tau}$ should be key variables to guide and design vision augmentation systems.

Given that the first description of tau-guided motion for rotary wing flight was for manuevers performed near to the ground, it seems reasonable to search for similar guidance mechanisms in fixed-wing flight close to the ground. Indeed, for up-and-away flight, transport aircraft pilots rely primarily on instruments for their flight information, as there is very little visual information available to them from outside the cockpit windows; tauists would also argue that tau is used to close aircraft motion gaps indicated by instruments if flown manually, i.e., the gap between the indicator's current and desired position is closed using tau motion strategies, e.g., the closure of the gap between the glide-slope needle and the desired glide-slope position during a glide-slope capture maneuver. This is not the subject of the current research but would make an interesting study. Much effort has been devoted to the analysis of the approach and land maneuver. This phase of flight is therefore a useful starting point for the search for fixed-wing tau-guidance strategies. Any parameters found in use to perform this maneuver may then form the basis for a display that recreates the optical cues that the pilot is using.

C. Spatial Gaps for the Flare Maneuver

To perform a search for tau-guided motion in the flare, a number of questions first need to be answered:

1) What gaps are being controlled by the pilot during the flare maneuver (for the purposes of this paper, the gaps will be restricted to those that might be sensed optically)?

2) If there is scope for controlling more than one gap during the flare maneuver, what gap pairs would it be sensible to control simultaneously?

3) If the pilot is controlling multiple gap closures, is the pilot coupling the taus of those gaps?

4) If one spatial gap is being used, is this being coupled with an intrinsic tau guide?

There are a number of potential spatial gaps that can be closed during a flare maneuver. A small sample of these are shown in Figs. 4 and 5, namely, 1) the height of the aircraft above the runway surface from the start of the flare to touchdown, 2) the vertical speed of the aircraft above the runway surface from the start of the flare to touchdown, 3) the change in aircraft pitch angle from θ_{appr} to θ_{td} during the flare, and 4) the slant range from the current aircraft position to the touchdown point (D_L in Fig. 5).

As far as slant range and angle are concerned, Lee [10] provides a hypothesis that a strategy of tau coupling between the perceived distance to the landing point to some distance ahead of the landing point is used. This situation is shown in Fig. 5, and is considered in detail in [10]. The hypothesis is extended in the reference to be applicable to an aircraft flare and landing. Given the current status of the research, this paper only deals with the first of the four spatial-gap closure possibilities and the investigation into its potential coupling with the general intrinsic tau guide. The remainder of the possibilities will be reported at a later date.

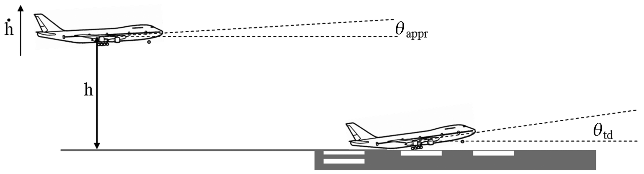


Fig. 4 Selection of spatial gaps to be closed for approach and flare maneuver.

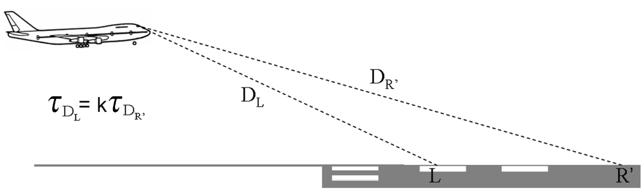


Fig. 5 Example tau-coupling hypothesis for approach and land maneuver.

III. Experimental Search for Tau-Based Guidance Strategies for the Flare

A number of analyses were carried out to investigate how the tau of the aircraft height changed over the duration of the maneuver. More specifically, the following simple hypothesis was tested:

$$\dot{\tau}_h = c \tag{8}$$

i.e., in a similar manner to drivers braking to a halt and helicopter pilots decelerating to a hover, fixed-wing pilots arrest the descent of the aircraft during the flare using a constant rate of change of tau (height) strategy. Two different approaches were taken for this investigation. First, historic flight-test data for the flare and landing were extracted from [18] and converted to the tau domain (from the phase-plane format, in which they were presented). Second, a simulated flight trial was conducted at the University of Liverpool to establish how (and if) the observed tau relationships changed as the visual conditions were degraded. The following section reports on the results of those analyses.

A. Flight-Test Experimental Data: Background

Before any simulator trials being run at the University of Liverpool, an analysis was performed on data extracted from [18]. The behavior of experienced pilots transitioning to the McDonnell-Douglas DC-10 aircraft had already been analyzed in the reference. A proportion of the pilots had been transitioned using DC-10 simulators whereas the remainder had performed the transition on the actual aircraft type. It was found that the simulator-trained pilots had a slightly inferior landing technique that was carried through to their check-rides in the real aircraft that led to them making heavy or inconsistent landings. Heffley et al. [18] analyzed this behavior using a pilot-in-the-loop model of the landing maneuver. It was found that the simulator-trained pilots exhibited a larger effective lag in commanding the flare.

From an ego-motion perspective, it might be said that the simulator-trained pilots were either unable to pick up suitable motion-gap closure cues or picked up erroneous motion-gap closure cues from the simulator displays that were then carried across to the real aircraft. It was therefore considered instructive to consider the motion gaps that could be discerned from the data available for the various groups of pilots defined in the reference.

B. Preliminary Analysis of Flight-Test Data

Figure 6 shows a representative sample of the results obtained from the analysis performed on the flight-test data. The "Group" label on the figure relates to how the pilots were divided in [18], namely, flight-trained pilots with check ride landings consistently less than 5 ft/s vertical touchdown velocity (FA); simulator-trained pilots with check ride landings consistently less than 5 ft/s (SA);

flight-trained pilots with check ride landings harder than 5 ft/s or height misjudgement tendencies (FC); and simulator-trained pilots with inconsistent check rides, i.e., no discernible improvement (SC). The pilot's number relates to the individual pilot in question and the check ride number indicates which of the three check rides used for data acquisition is displayed. A fifth group is defined in the reference (group SB: simulator-trained pilots where the first check ride landing was harder than 5 ft/s but followed by continual improvement). These latter data have not been investigated as they were given the lowest priority in the limited time available to conduct the analyses.

From this initial investigation, it appeared that the constant rate of change of tau (height) hypothesis, given in Eq. (8), was indeed correct during the last few seconds before touchdown. Two different techniques for the flare are immediately apparent:

1) Pilot 409: The pilot commences the flare and decelerates the aircraft to a vertical speed that is maintained to touchdown. One additional feature of this technique is that τ_h is held approximately constant for a period before touchdown.

2) Pilots 416 and 436: The pilot commences the flare and continues to decelerate the aircraft in the vertical axis (not necessarily at a constant rate) until touchdown. This results in an approach and flare that features a constant rate of change of τ_h [as per Eq. (2)]. In the case of pilot 416, the technique was adopted but at a much higher descent rate (and value of $\dot{\tau}_h$) resulting in a touchdown with a high vertical speed. This gives the pilot less time to assimilate the (arguably stronger) cues provided by the view of the outside world from the flare height.

C. Flight Simulation Experimental Data: Background

To be able to begin to design displays to recreate the visual cues that the pilot uses, it was considered necessary to check that the phenomena observed from the flight-test data could be recreated in the Bibby Flight Simulator [19] at the University of Liverpool. Once this could be established, the observation of how the phenomena changed in degraded visual conditions could begin.

An experiment was therefore performed whereby the pilot, a professional airline captain and former test pilot, was required to fly an approach to a runway in good visual conditions. If the trends observed in the flight-test data could be recreated, then a further series of approaches would be performed using a variety of degraded visual conditions. In this way, it might be expected that any relationships that exist for the proposed variables in good visual conditions will break down as the visibility is steadily degraded in a realistic manner.

The aircraft model used was one developed specifically for the project using ART's FLIGHTLAB software [19]. The model is a generic large transport model that is based upon publicly available data for the Boeing 707 aircraft.

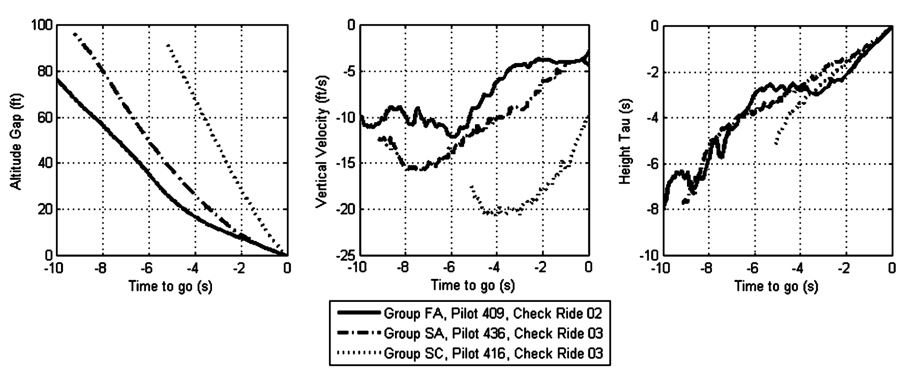
Figure 7 shows two examples of flares conducted in good visual conditions. It shows that the same pilot used both techniques observed in [18] to land in the same visual conditions. That is, on one occasion (run 81), a constant vertical velocity was maintained for the last moments of the flare ($\dot{\tau}_h = 1.0$), whereas on the second occasion (run 90), the vertical speed was constantly being reduced to the point of touchdown. Run 81 also shows the constant tau element of the maneuver very clearly, 3–4 s before touchdown. The techniques to flare and land a fixed-wing aircraft observed in flight-test data were, therefore, recreated. This provides a good level of confidence in both the aircraft model being used and in any results achieved from observations made in simulated flight in degraded visual conditions.

Once satisfied that the simulation environment was a valid one, a further experiment was performed to establish 1) how (and if) the variation of τ_h changed when the visual cues received by the pilot were obscured from view, and 2) how (and if) the variation of τ_h changed with piloting experience.

Four male pilots were used for the experiment. Table 1 lists the pilot identifiers and their corresponding levels of experience. The list is provided in descending order of flight experience.

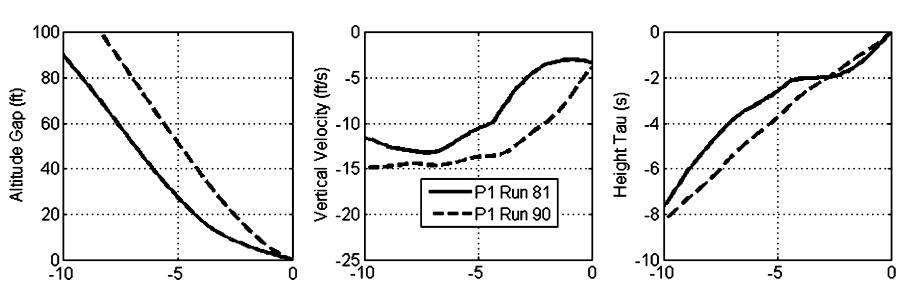
The simulated degraded visual condition images were generated using a system developed by BAE SYSTEMS (known as "Landscape") that uses SGI's OpenGL Performer software as its basis [20]. Visibility was restricted using the fog model that Landscape provides. The visual conditions used during the experiment are given in Table 2 and a small sample of these is shown in Fig. 8.

The "Categories" given in Table 2 refer to the aircraft operating minima for making an approach and landing to a runway. The



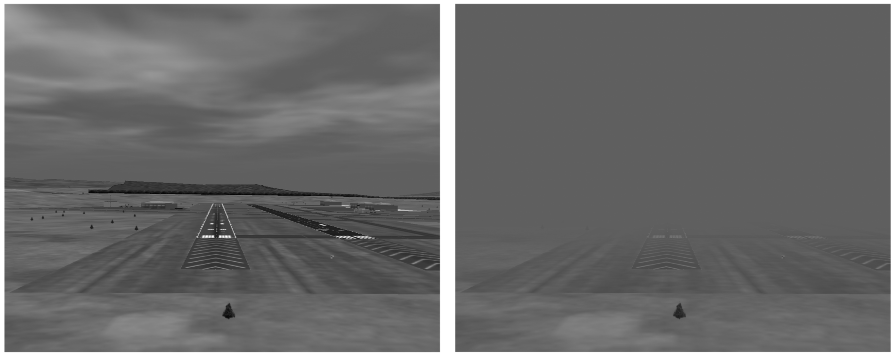
a) b) c)

Fig. 6 Sample flight-test data: a) height gap, b) vertical speed, and c) height tau τ_h.



a) b) c)

Fig. 7 Example flare simulation validation results.



a) b)

Fig. 8 Simulated view on final approach for a) good (V1) and b) degraded (V4) visual environments.

runway visual range (RVR) values used for the trial test points were taken from [2].

"Rich" scene content refers to the use of a highly detailed outside world database in terms of both texture and objects in the scene (Fig. 8a). Of course, for the degraded visual condition test cases, this texture and object detail is obscured by fog (Fig. 8b). The "impoverished" scene used the same database but was not lit, so all objects appeared black. The only objects visible to the pilots were a set of runway lights that provided an outline of the runway and that also marked its centerline.

The initial condition for the aircraft was set at two runway lengths from the touchdown aiming point of a simulated runway in a trimmed 3.5 deg glide-slope descent at $1.2V_{\text{stall}}$. Each pilot was instructed to maintain the approach condition until an appropriate height to commence the flare was reached and was requested to touch down in the marked runway touchdown zone. For tests in which the visibility was reduced such that the runway was not visible at the start of the maneuver, instrument landing system (ILS) guidance was provided on the simulator instrumentation. The lateral states of the aircraft model (e.g., roll angle, heading angle, etc.) were locked out, giving the pilots a control task to perform in the longitudinal/vertical plane only.

D. Analysis of Flight in a Degraded Visual Environment

Figure 9 shows typical results for pilot P1's flare maneuvers for a small selection of the visual conditions listed in Table 2. In good visual conditions (run 90), the pilot demonstrates a decelerating approach to the point of touchdown as already described. As the visibility is decreased to 1800 ft (V5), a nonlinearity is introduced to the τ_h profile as the pilot checks the descent rate and then allows it to increase to touchdown. For the test case where the visibility is reduced further (V6b), a discontinuity is introduced to the τ_h profile before touchdown. In this case, the descent rate is arrested and the aircraft allowed to fly parallel or even away from the runway surface for a short period until it sinks onto the runway surface. It is evident from the piloting technique involved in these cases that a larger than normal initial control input is made. This is interpreted, following discussions with the test pilot, as the pilot becoming visual with the runway at a later stage than would normally be desirable, i.e., well below the usual height at which a decision to land the aircraft manually would be made and entering a control input to ensure that the aircraft does not strike the ground. Once visual with the outside world and comfortable with the aircraft flight path, the pilot then allows the speed to decay to bring the aircraft into contact with the runway surface.

The longitudinal control inputs made for these cases are shown in Fig. 10, comparing equivalent inputs made with the good and poor visual condition runs. It can be seen that for the good visibility case, a progressive longitudinal stick input is made. For the degraded visibility case, a number of rapid control inputs are made. The pilot also commented that for a real in-flight situation, this technique is a valid one unless the runway length is limiting, i.e., the runway length available is only just sufficient for the calculated safe landing distance required. Of course, the point should be made that in a real operational environment, based upon discussions with pilot P1, a landing in the simulated visual conditions would most likely have been conducted using an automatic landing system. For the purposes of this paper, flares that correspond to the form of run 125 in Fig. 9 shall be referred to as "balloons." (A ballooned landing is one where the pitch-up to flare is "overdone" and a reduced descent rate turns into an ascent rate. It is this crossing of the zero vertical velocity boundary that causes the observed tau discontinuity.)

Figure 11 shows two specific examples of a markedly different approach profile to those already discussed for the cases where visibility or the scene content is extremely reduced. In these cases, $\dot{\tau}_h$ is constant but with a value close to 1. Both cases are associated with approximately constant and high touchdown vertical velocities. The touchdown velocity is approximately the same as that that would be

Table 1 Details of pilots used for degraded visual conditions experiment

Pilot identification	Level of experience
P1	Professional Airline Captain, former test pilot, former Royal Navy pilot, high level of experience in Liverpool simulator
P2	Lapsed private pilot, experienced simulator pilot
P3	Moderately experienced simulator pilot
P4	Small amount of experience as simulator pilot

Table 2 Visibility conditions used for simulated flight experiments

Visibility condition	Visibility, ft	Day/night	Cloud base, ft	Comment	Scene content
V1	Unlimited	Day	——	Baseline condition	Rich
V2	Unlimited	Night	——	Runway lights only	Impoverished
V3	8000	Day	——	Visual range = one runway length	Rich
V4	4000	Day	——	Visual range = half runway length	Rich
V5	1800	Day	——	Equivalent category I RVR	Rich
V5b	1800	Day	200	Equivalent category I RVR	Rich
V6	700	Day	——	Equivalent category IIIa RVR	Rich
V6b	700	Day	50	Equivalent category IIIa RVR	Rich
V7	150	Day	——	Equivalent category IIIb RVR	Rich

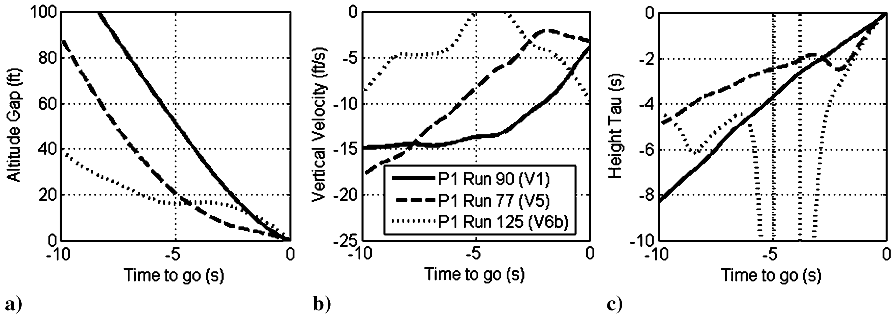


Fig. 9 Comparison of simulated flare maneuver trajectories as visual conditions are degraded.

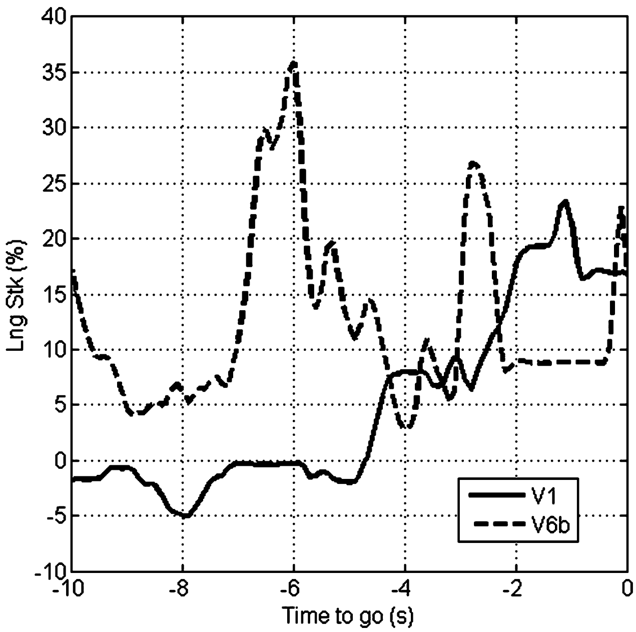


Fig. 10 Comparison of simulated flare longitudinal control inputs in good (V1) and extremely degraded (V6b) visual conditions.

achieved by following the ILS glide-slope indicator into the runway surface (140 knots for a 3.5 deg glide slope requires a descent rate of 14.4 ft/s). In the first case (V2), this implies that the pilot has not been able to detect sufficient visual cues to check his descent rate from that indicated by the ILS before impact with the ground surface. For the second case, the pilot did not see the runway surface until the aircraft had touched down. This did not happen for all flares conducted at this visibility level. In this particular case, the pilot became visual with the runway surface between the white runway centerlines. The runway surface and fog color are similar so the pilot was unable to detect the impending collision (and, by implication, had been using the centerlines as a cue for proximity to the runway surface in the extremely degraded visibility).

Overall, the experimental results indicate that the coherent relationship that exists between τ_h and time as an appropriate flare maneuver is executed in good visual conditions can break down when the pilot gains sight of the runway at a late stage and has to take late evasive action. The coherence returns when no or little pilot input is made to flare the aircraft, and, for a professional pilot at least, this only occurs when limited or no visual cues are available. In these cases, the height-tau profile is indicative of an approximately constant velocity or even accelerative approach to the runway surface (at or around the descent rate commanded by the ILS).

E. Analysis of Flight with Varying Pilot Experience

A number of approaches to touchdown were conducted using the pilots of Table 1 in the visual conditions of Table 2. To try to understand the differences between the various groups of pilots, an analysis was carried out of the slopes of the τ_h curves (giving $\dot{\tau}_h$) close to touchdown, and this was correlated with the velocity at touchdown (bearing in mind that a touchdown is considered "acceptable," for the purposes of this paper, if the descent rate is less than 5 ft/s [18]). This analysis is shown in Fig. 12.

There are three reasonably distinct groups of data apparent in Fig. 12a. The first of these are the small number of data points clustered around $\dot{\tau}_h$ of 1 at low to moderate touchdown velocities (-2.0 to -7.0 ft/s approx.). These are marked "A" in the figure. A rate of change of τ of 1.0 implies a constant (in this case vertical) velocity. These data are indeed associated with the constant velocity flare technique described earlier in the paper. The second group, marked "B" in the figure, are those data that fall into an approximately linear relationship between $\dot{\tau}_h$ and touchdown velocity, ranging from very low touchdown velocities at $\dot{\tau}_h$ of 0.6 to high touchdown velocities at $\dot{\tau}_h$ of 1.1. These data are associated with the continuous deceleration landing technique described earlier.

Tau theory states that such values of $\dot{\tau}_h$, i.e., those values evident in Group B, are associated with a "collision" with a surface. This is entirely consistent with piloting technique whereby transport aircraft are generally flown positively onto a runway (rather then the classical notion of flaring to round out to fly parallel to the runway surface before sinking to the ground. A slight misjudgment using this technique in a transport aircraft can result in a significant float down a runway). The higher the value of $\dot{\tau}_h$, the greater is the severity of that collision and the higher the touchdown velocity, consistent with tau

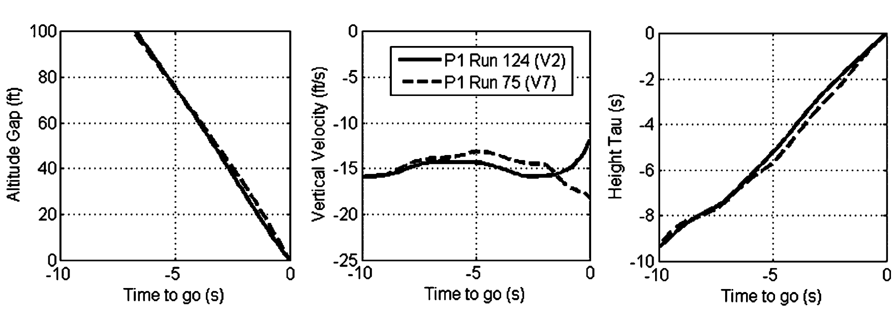
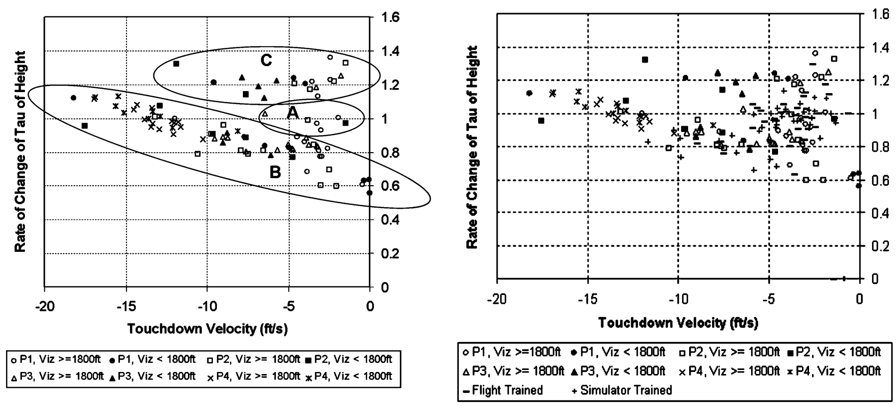


Fig. 11 Flare trajectories in extremely degraded visual conditions.



a) b)

Fig. 12 Variation of height tau-dot with touchdown velocity: a) simulation data only and b) simulation and flight-test data.

theory. The final group are those data clustered around a $\dot{\tau}_h$ of 1.2 (implying an accelerative approach to a surface). These data, marked "C" in Fig. 12, are associated with those touchdowns where the tau relationship has broken down before touchdown, as described earlier in the paper. An accelerative contact with the runway is consistent with a ballooned landing because the aircraft has, a few seconds previously, been ascending and has had to accelerate downwards to reach the contact the runway. The pilot will be trying to get the aircraft on the ground and may not fully control the accelerative motion or may even allow it to occur to get the aircraft down onto the runway.

In terms of pilot experience, the professional pilot (pilot P1), with a few notable exceptions, maintains low touchdown velocities and has used all three techniques to land the aircraft. There is little to choose between the more experienced "amateur" pilots (pilots P2 and P3), both demonstrating a preference for the continuous deceleration technique when not having to deal with ballooned landings. These pilots exhibit the selection of slightly larger coupling constants for this technique than pilot P1. Pilot P3 shows his inexperience with a cluster of flares around a $\dot{\tau}_h$ of 1.0. This is a consequence of pilot P3 failing to flare to any degree and making contact with the runway at the descent rate commanded by the ILS.

Figure 11b adds further data points in the form of the flight-test data. Now, Heffley et al. [18] do not provide associated data on the experience of the individual pilots involved or of the visual conditions in which they flew, so a direct comparison is not strictly possible. What is clear is that these pilots appear to have a preference for the constant velocity touchdown technique rather than the continuous deceleration flare technique.

F. Intrinsic Tau Guidance

Inspection of the height and velocity profiles for those flares that exhibit the continuous deceleration technique reveals a similarity in shape to the intrinsic tau-guide curves of Fig. 3 over the final stage of the landing ($t = -0.5$ to 0.0 s in Fig. 3). Pilots are taught, very early in their training, to "recognize the picture" in the windscreen for many aspects of flight. These include reference pitch attitudes for both climbing and level flight and, of course, the height from which to flare. How this picture changes during the course of the flare will influence the pilot control inputs to guide the aircraft to the ground safely. It was hypothesized that training to "remember the picture" initialized an intrinsic tau guide that the pilot could then use to flare the aircraft as per the training and subsequent experience gained. It was therefore considered appropriate to further explore the data from [18] and the simulated flight-test data that conformed to the continuous deceleration technique in the context of tau coupling with the intrinsic tau guide of Eq. (6). For the analysis, to be able to calculate T, a start point has to be determined for the flare. The flare was considered to have started when the vertical velocity had fallen to 90% of its peak (negative) value during the approach. T would then be the time to touchdown. Figure 13 shows typical results for this analysis.

A number of features stand out in Fig. 13. It can be seen in all cases that over the period of the flare (the final 5 s or so of the motion), the motion guide and tau guide have a strong correlation (the analysis was tuned to provide a correlation line when the R^2 value reached 0.97 or greater).

The second feature to note is that for the landings at an "acceptable" vertical velocity (pilot 436 and P1), the intrinsic tau guide remains greater (more negative) than, but close to, the aircraft tau until the coupling finally occurs in the last few seconds before touchdown. The reverse is true for the hard landing case (pilot 416). It would therefore appear to be important the way in which the intrinsic coupling is approached. Maintaining the aircraft tau close to but at a lower value than the intrinsic tau guide appears to be a more successful strategy to achieve a safe touchdown than the reverse. Padfield et al. [3] liken the intrinsic tau-guide to a ball, from which the pilot views the aircraft, forming a mental image of its flight path. If the pilot is able to maintain the ball/viewing point, metaphorically speaking, above the aircraft, s/he will have a good view of what the aircraft is doing and can therefore maintain the correct control inputs to achieve a safe landing. In the case where the aircraft tau is greater than the intrinsic tau guide, the pilot's mental viewpoint is below the aircraft and so s/he is able to form a less reliable view of the aircraft's prospective flight path. It is a truism that pilots prefer to "stay ahead of the aircraft" to fly safely. It would appear that, metaphorically speaking at least, the reverse is actually true from a tau-guidance perspective.

Figure 14 presents a summary of the analysis of the coupling constant achieved between the motion and guidance tau values for those flare maneuvers where the continuous deceleration technique was used (professional pilots only in this case). It shows a roughly linear relationship between the coupling constant used by the pilots and the touchdown velocity achieved. The relationship appears to suggest that the higher the constant used by the pilot (to couple the tau of the height of the aircraft to the intrinsic tau guide), the higher is the touchdown velocity (as would be expected by inspection of Fig. 3). By the very nature of the grouping of the pilots, the simulator-trained pilots have a tendency, for some flare cases, to use larger coupling constants than those of their flight-trained colleagues.

The Liverpool test points are consistent with this trend. They show that reasonable touchdown rates (<5 ft/s) were achieved in all levels of visual environment. On a few occasions, high touchdown rates and coupling constants are observed. The data appear to show that high touchdown rates were achieved more often in good visual environments (visibility $\geq$ 1800 ft) than in degraded environments. These test points (marked "A") were actually conducted in a simulated night scene where the only cues available to the pilot were the runway lights. The visual scene, in this case, was highly degraded (but with good visibility, there just wasn't much to see). It is believed that, in these instances, the cues from the outside world visual scene were insufficient to allow the pilot to detect the high sink rate that had developed.

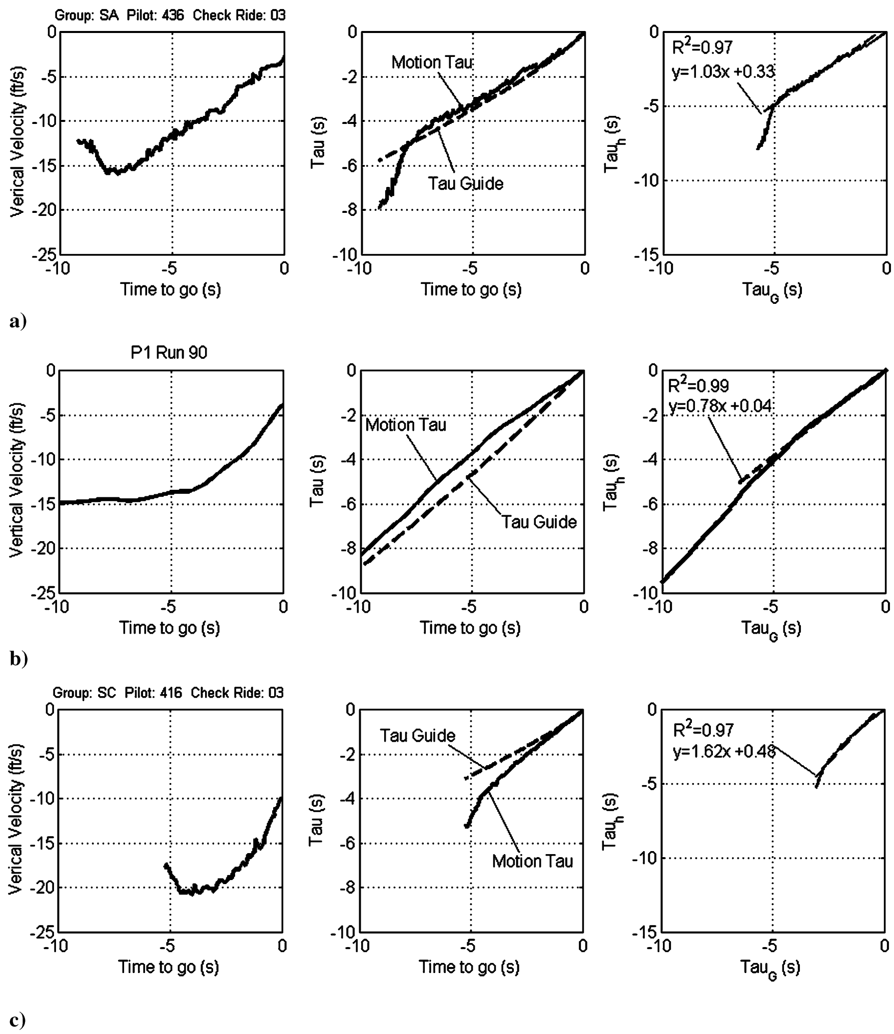


Fig. 13 Example tau-guide coupling analyses: a) and b) constant deceleration to an acceptable touchdown velocity, and c) deceleration to an unacceptably high touchdown velocity.

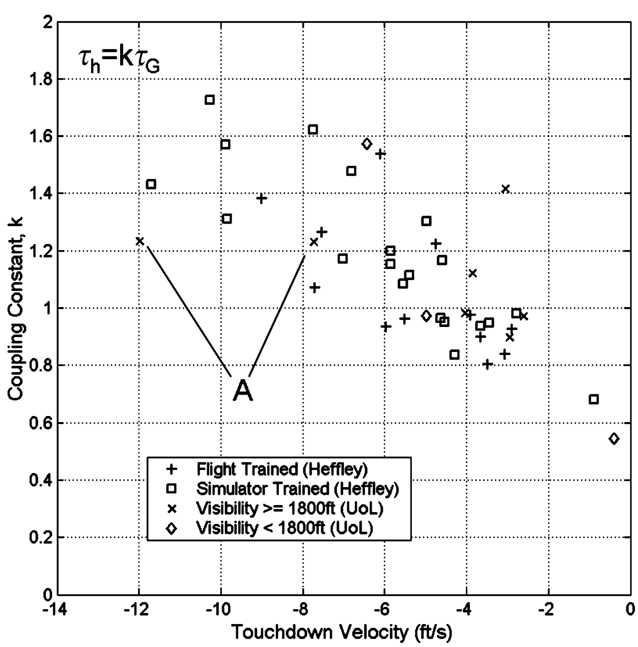


Fig. 14 Variation of general intrinsic tau guide coupling constant with vertical touchdown velocity.

Overall, the data of Fig. 14 suggests that control of touchdown velocity may be achieved by providing the pilot with a means of coupling the aircraft motion onto the general intrinsic tau guide with a suitable value of the coupling constant k. The results given in Fig. 14 show that a value of k between 0.6 and 0.8 would be suitable. These would provide a positive touchdown but with an acceptable vertical velocity.

IV. Discussion of Results

A number of points raised during the course of this paper are worthy of further discussion. The first of these concerns the ability of an analysis in the tau domain to identify the type of trajectory being flown. A constant tau strategy implies an exponential flight path whereas a constant $\dot{\tau}_h$ strategy implies a power law flight path (see Appendices B and E). The corollary to this, in the context of the present work, is that the use of tau provides a simple mathematical model to allow the definition of a particular flight path to close a particular gap in a controlled manner. This opens up the possibility for designing a command-type display whereby the pilot guides the aircraft motion-tau in response to a command indicator that is being guided either by a constant or intrinsic tau-guide control law.

A second, but related, point is that for landings carried out using the constant tau followed by constant velocity touchdown technique, the constant value of tau selected tends to be in the region of 3.0 s for the flight-test data and 2.0 s for the simulated flight-test data. These values are both approximately 1.6 times the heave mode time constant of the transport aircraft involved. (The time constant for the DC-10 aircraft of [14] was estimated to be 1.8 s. For the transport aircraft simulation model developed for this work, the heave constant has been estimated at 1.3 s for the touchdown configuration.) It is interesting to speculate that in these cases, the pilot is reducing the rate of descent in a manner that allows enough time to be able to correct the flight path of the aircraft should any errors in the required trajectory occur. At the present time, this is purely conjecture and further experimentation is required to test such a hypothesis, but the idea that there might be a relationship between the maneuver time and the aircraft response time is rather intuitive and has been picked up in earlier research [21].

It is recognized that there are several limitations with the work carried out so far. The first of these is, of course, that only one professional pilot has been used thus far in the simulated flight trials. More trials are planned with a number of current pilots to check that the trends observed are valid. To move this work forward in the simulation environment, data from a larger pilot population will be required.

The second limitation of the simulation work carried out so far is that, whereas the best efforts have been made to provide the pilot with only outside world visual cues to allow him to make judgements about the landing flare, this has not always been possible. Whilst the pilot was not provided with altimeters, it was necessary to provide ILS indications on a horizontal situation indicator (HSI) in order that the runway could be located in the degraded visibility test cases. This instrument provided two additional cues that the touchdown zone of the runway was close by (and that a flare was required imminently). Firstly, the HSI used gave an indication of distance to go to the ILS beacon. As this counted down towards zero, provided that the aircraft was on or around the correct glide slope, the pilot was alerted to the fact that the runway surface was being approached. The second HSI indication of an imminent arrival at the runway surface was the ILS glide-slope indicator itself. As per the real system, the glide-slope deviation indicator becomes increasingly sensitive as the distance to the ILS beacon decreases. This increase in sensitivity gave the pilots a cue that the touchdown zone was approaching. It is not considered that these additional cues have had an adverse effect on the results presented. Whilst both gave a realistic indication to the pilot that the runway surface was approaching, they did not provide any flare or touchdown guidance. This was left to the pilot based upon his view of the outside world.

The simulation results reported in this paper have all been conducted in "ideal" atmospheric conditions, i.e., no wind, no gusts, and where the pilot has only had to close vertical motion gaps. In a real landing, the pilot will have many gaps to close. Further work needs to be performed to establish the requirements on tau coupling when moving more freely in 3-D world and what a tau-based display would look like to assist in this procedure.

Limitations aside, the tau of height provides a potentially exciting opportunity in a number of areas. The first of these is the one that initiated the work reported in this paper: the development of aircraft displays. A command-type display has already been mentioned. Display symbology could be driven directly, for example, using τ_h to force the pilot to adopt a particular strategy to ensure an appropriate landing touchdown velocity. Alternatively, displays could be developed that stimulate tau indirectly (presenting the pilot with a means to establish the motion of the outside world might be an example of this). Such displays, developed for aircraft use during approach and land would then be assessed in terms of their ability to generate suitable tau-guide coupling constants or constant $\dot{\tau}$ responses. The next phase of this research will be to assess the feasibility of both of these approaches to display design.

The preceding term "appropriate touchdown velocity" could be replaced with "safe touchdown velocity." Flight safety is another area where the optical variable tau might find a use. Major airlines already monitor their operations using sensors onboard the aircraft. These sensors trigger "events" when a particular condition is met, e.g., for one UK-based operator, a high rate of descent below 2000 ft triggers a monitoring event. In the context of this paper, examples of events that might trigger an alert event are the use of a height-tau coupling constant above a predefined value or a $\dot{\tau}_h$ of, say, between 0.75 and 0.95.

A third potential use of the parameter tau would be in flight training. The landing of an aircraft is perhaps one of the more difficult maneuvers to achieve and the progress of a student's ability to perform it well could be measured in terms of the height-tau couplings achieved or the $\dot{\tau}_h$ values used. There is perhaps even an argument that pilot candidate selection could be strengthened through the use of tau. It might be, for instance, that those candidates that show an aptitude for piloting an aircraft are the ones that show an early ability to adopt safe tau-based strategies in their flying; interestingly, this concept takes us back to the early work performed by Gibson on optical flow. Further work would need to be carried out to establish any validity to this hypothesis of course.

Overall, this paper proposes that an alternative, temporal parameter tau, which has evidence for its existence in many examples in the natural world, may also be used by pilots controlling the height of their aircraft above ground during the landing flare maneuver.

V. Conclusion

An analysis has been performed on both flight-test and simulated flight-test flare data for large transport fixed-wing aircraft to establish whether or not guidance strategies exist using the optical variable "tau" (time to contact surface at current closure rate). Tau requires that some form of sensory gap be closed. This paper reported on the closure of the spatial-gap "height" as the aircraft is brought into contact with the runway surface. An analysis has been carried out to test the hypothesis that the rate of change of the tau of height remains constant during the flare maneuver. The evidence presented supports this hypothesis. Furthermore, it has been found that two techniques are evident for the flare maneuver in terms of rate if change of τ_h for the flare:

1) Pilot selects and holds $\dot{\tau}_h$ equal to unity until the aircraft contacts the runway surface. This is equivalent to establishing and maintaining a constant vertical velocity to touchdown. In some cases, τ_h is maintained at a constant value (i.e., $\dot{\tau}_h = 0$) shortly before touchdown.

2) Pilot selects a rate of change of τ_h at a constant value less than 1 but greater than 0.6 before touchdown. The size of this constant $\dot{\tau}_h$ selected by the pilot directly influences the vertical velocity achieved at touchdown.

For the second flare technique, a correlation has been found between the tau of height τ_h and the general intrinsic tau guide τ_G for the flare. It has been shown that a criterion for a successful landing (judged in terms of touchdown rate) is the direction from which the aircraft tau approaches the tau guide. Conceptually, the pilot must maintain a mental model of the aircraft trajectory, viewed from above the aircraft, to touch down at a reasonable vertical speed.

Degradation of the visual conditions in which simulated flare maneuvers were carried out modified the τ_h profiles during the landing. Approach and landings carried out in good visual conditions provided linear relationships between τ_h and time and τ_h and τ_G. When visual conditions were degraded to levels normally associated with automated landing systems being used, highly nonlinear height-tau profiles were observed for manually flown touchdowns. These were usually associated with the pilot reacting aggressively to the late acquisition of visual cues to prevent a hard touchdown followed by a controlled descent to the runway surface. When the runway was not seen or minimal cues were available in the visual scene, $\dot{\tau}_h$ values associated with constant velocity or accelerative contact with the runway surface were observed.

An investigation into the variation of the observed tau-based strategies with pilot experience showed that even pilots of limited experience demonstrate tau-based strategies for the flare and landing. Using a pilot with very few hours' experience proved little use as the aircraft was not flared, but flown into the ground at the vertical descent rate commanded by the ILS.

This work has been carried out with the aim of developing guidelines for the design of novel display technology. The next phase of the work is to design display symbology that can make use of the reported τ_h information to enable pilots to land fixed-wing aircraft in visual conditions that would otherwise require automated landing systems to be present on both aircraft and airfield.

Appendix A: Sensing Height Tau from Optic Flow

Lee [10] shows that if two taus are coupled, then they are related by a power law relationship. Using the definition of tau in Eq. (1),

$$\frac{u}{\dot{u}} = K\frac{v}{\dot{v}} \tag{A1}$$

Inverting Eq. (A1) and integrating with respect to time gives

$$\ln u = \left(\frac{1}{K}\right)\ln v + \ln C \tag{A2}$$

Therefore

$$u = Cv^{1/K} \tag{A3}$$

Equation (A3) shows the power law relationship.

Now, Lee [16] shows how a motion tau can be sensed directly from the optic flowfield created when an observer, O, moves within its environment. This paper is concerned primarily with vertical motion so the discussion will be restricted to movement in this axis. Furthermore, for the sake of simplicity, only monocular vision is considered. The projection plane, i.e., retina, is also considered to be flat. Figure A1 shows an observer traveling in the Z direction at a velocity dZ/dt parallel to a vertical plane. A point P makes up part of the image of that plane on the projection plane at point P'. From similar triangles we have

$$\frac{z_{p'}}{d} = \frac{Z_p}{X_p} \tag{A4}$$

When O is moving at dZ/dt, d and X_P are constant, so

$$z_{p'} = \frac{d}{X_p}Z_p \tag{A5}$$

Equation (A5) is equivalent to Eq. (A3) with $C = (d/X_P)$ and $K = 1$. We can therefore formally say that

$$\tau_{ZP'} = \tau_{ZP} \tag{A6}$$

i.e., a sensory flowfield tau is coupled to an externally perceived tau. In this way, information picked up by the pilot's eyes from a vertical motion can be converted into information about vertical gaps being closed.

Appendix B: Constant Rate of Change of Tau Motion

If a constant rate of change of height tau c is assumed,

$$\frac{d}{dt}\left(\frac{h}{\dot{h}}\right) = c \tag{B1}$$

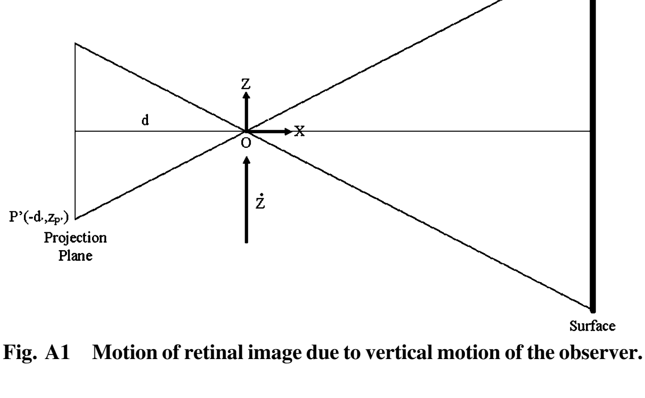


Fig. A1 Motion of retinal image due to vertical motion of the observer.

where h is current height gap to be closed and $\dot{h}$ is the instantaneous rate of closure of that gap. Integrating both sides with respect to time, bearing in mind that, by convention, at time $t = 0$, $\tau_h = 0$,

$$\frac{h}{\dot{h}} = ct \tag{B2}$$

Rearranging Eq. (B6) yields

$$\frac{c}{h}\,dh = \frac{1}{t}\,dt \tag{B3}$$

Evaluating the integrals and rearranging gives

$$\ln (h^c) = \ln (Ct) \tag{B4}$$

To find the constant of integration C, at $t = -T$, maneuver duration, $h = h_0$, height at commencement of flare,

$$h = -\left[\frac{h_0^c}{T}t\right]^{1/c} \tag{B5}$$

Calculation of the instantaneous vertical velocity can now be made by differentiating Eq. (B5):

$$\dot{h} = -\left[\frac{h_0}{cT^{1/c}}t^{(1-c/c)}\right] \tag{B6}$$

Appendix C: Intrinsic Tau Guide

Lee in Reiser et al. [17] derives the defining equation for the general intrinsic tau guide, and the resulting motion that is implied by coupling onto such a guide as follows:

$$\tau_G = \frac{t(T+t)}{T+2t} \qquad -T \le t \le 0 \tag{C1}$$

To normalize Eq. (C1), define $t_n = t/T_G$, giving

$$\tau_G = \frac{T_G(1+t_n)t_n}{1+2t_n} \tag{C2}$$

If a body under motion, closing a gap x, is coupled onto a general intrinsic tau guide then

$$\tau_x = \frac{x}{\dot{x}} = k\frac{T_G(1+t_n)t_n}{1+2t_n} \tag{C3}$$

Integrating Eq. (C3) with respect to time successively yields

$$x = x_m 2^{2/k}(t_n - t_n^2)^{1/k} \tag{C4}$$

and:

$$\dot{x} = \frac{x_m}{T_G}\frac{2^{2/k}}{k}(-1-2t_n)(t_n - t_n^2)^{(1/k)-1} \tag{C5}$$

where x_m is the minimum value attained by x.

Appendix D: Equivalence of τ_g over Last Half of a τ_G Motion

Padfield et al. [3] and Lee[10] provide the defining equation for the original (constant acceleration) tau guide, viz

$$\tau_g = \frac{1}{2}\left(t - \frac{T^2}{t}\right) \qquad 0 < t \le T \tag{D1}$$

Reiser et al. [17] provide the defining equation for the general intrinsic tau guide, viz

$$\tau_G = \frac{t(T+t)}{T+2t} \qquad -T \le t \le 0 \tag{D2}$$

Now, τ_g is equivalent to τ_G for the last half of a τ_G motion. For this period of the motion,

$$T_g = \frac{T_G}{2} \tag{D3}$$

and

$$t_g = t_G + \frac{T_G}{2} \tag{D4}$$

where T_g is the duration of the motion guided by τ_g, t_g is the current time during this motion, T_G is the duration of the equivalent motion guided by τ_G, and t_G is the current time during this motion. Substituting (D3) and (D4) into (D1) (with $t = t_g$ and $T = T_g$),

$$\tau_g = \frac{1}{2}\left(\frac{[(2t_G + T_G)/2]^2 - (T_G/2)^2}{(2t_G + T_G)/2}\right) \tag{D5}$$

and hence

$$\tau_g = \frac{t_G(t_G + T_G)}{T_G + 2t_G} \tag{D6}$$

Equation (D6) is now in the same form as Eq. (D2).

Appendix E: Implication of Constant Height Tau and Height Tau-Dot on Flight Path

If τ_h is kept constant, then it can be shown that an exponential flight path is flown with respect to time as follows: If

$$\tau_h = a \tag{E1}$$

then

$$\frac{h}{\dot{h}} = a \tag{E2}$$

Rearranging Eq. (E2) and writing it in longhand form gives

$$\frac{\mathrm{d}h}{\mathrm{d}t} - \frac{1}{a}h = 0 \tag{E3}$$

The solution for which is

$$h = Ce^{t/a} \tag{E4}$$

where C is the integration constant (which will vary with the initial conditions as the pilot initiates the exponential phase of the flare) and a is the constant value of τ_h adopted for the maneuver. From Eq. (E4), it can be seen that during a constant tau phase of flight, the aircraft height (and hence vertical speed) is an exponential function of time.

Acknowledgments

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